Control-flow Guided Property Directed Reachability for Imperative Synchronous Programs

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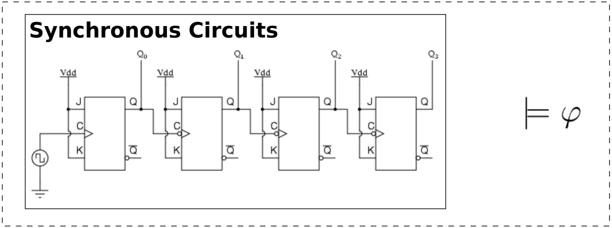
Table of Contents

Outline

1. Motivation
2. Property Directed Reachability
3. Control-flow Guided PDR for Imperative Synchronous Programs

Formal Verification of Synchronous Hardware Circuits

- PDR: a very efficient verification method based on induction



Formal Verification of Synchronous Programs

- PDR: a very efficient verification method based on induction

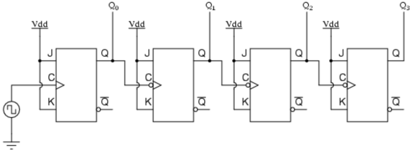
Synchronous Programs

```
module M(event bool ?a,?b,o1,o2) {
  loop {
    l1: pause;
      if(o1 & (a|b)) {
        emit(o2);
        l2: await(a);
      }
    }
}
```

$\models \varphi'$

synthesize

Synchronous Circuits



$\models \varphi$

Imperative Synchronous Programs

Imperative Synchronous Languages: e.g. Quartz

- macro steps: consumption of one logical time unit
- micro steps: no logical time consumption

$\Rightarrow$ synchronous reactive model of computation

Control-flow Information

- not needed for synthesis
- useful for formal verification

Goals

Target: Safety Property Verification of Imperative Synchronous Programs

- PDR: relies on good estimation of the reachable states

Our Heuristic: Improve it by Reachable Control-flow States Computation

- linear-time static analysis
- symbolic reachability analysis

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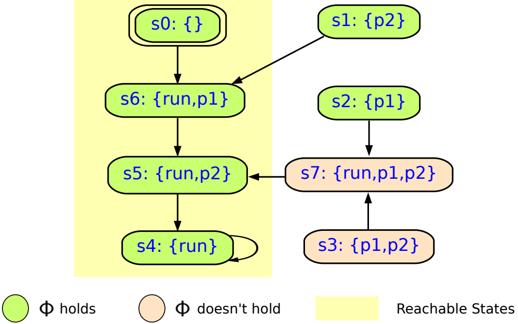
Safety Property Verification

Target: Prove Φ is valid w.r.t. $\mathcal{K}$

- a state transition system: $\mathcal{K} := (\mathcal{V}, \mathcal{I}, \mathcal{T})$
- a safety property: Φ
- Φ holds on all reachable states of $\mathcal{K}$




```
module CfSeq(){
    p1: pause;
    p2: pause;
}
```

$$
\begin{aligned}
\mathcal{V} &:= \{\mathrm{run}, \mathrm{p1}, \mathrm{p2}\} \\
\mathcal{I} &:= \neg(\mathrm{run} \vee \mathrm{p1} \vee \mathrm{p2}) \\
\mathcal{T} &:= \quad \mathbf{next}(\mathrm{run}) \leftrightarrow \mathbf{true} \\
&\quad \wedge (\mathbf{next}(\mathrm{p1}) \leftrightarrow \neg\mathrm{run}) \\
&\quad \wedge (\mathbf{next}(\mathrm{p2}) \leftrightarrow \mathrm{p1})
\end{aligned}
$$

$$\Phi := \neg(\mathrm{p1} \wedge \mathrm{p2})$$

Safety Property Verification by Induction

Target: Prove Φ is valid w.r.t. $\mathcal{K}$

- a state transition system: $\mathcal{K} := (\mathcal{V}, \mathcal{I}, \mathcal{T})$
- a safety property: Φ
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Φ is inductive w.r.t. $\mathcal{K}$

- induction base: Φ holds in all initial states
- induction step: Φ-states have no successor violating Φ

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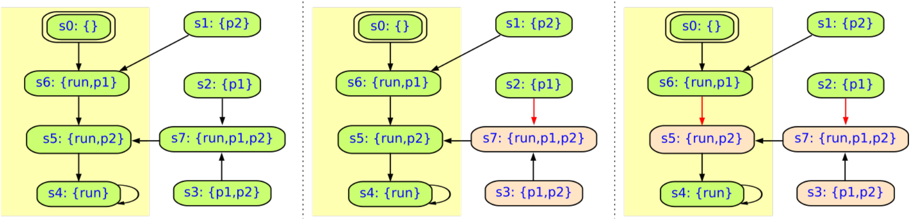
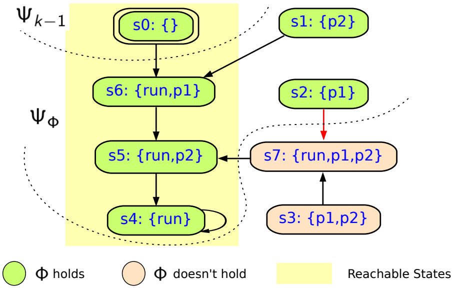


Property Directed Reachability

PDR method constructs a sequence of clause sets $\Psi_0, \ldots, \Psi_k$ that overapproximate the states reachable in $0, \ldots, k$ steps.

- incremental induction: extend the sequence $\Psi_0, \ldots, \Psi_k$
- unreachability checking:
  indentify counterexamples to induction (CTIs)



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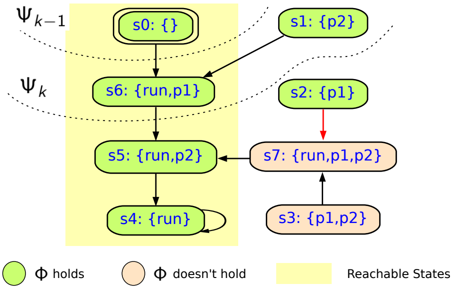

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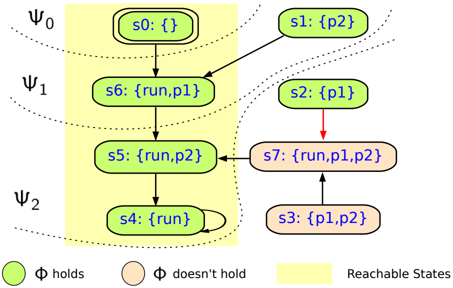

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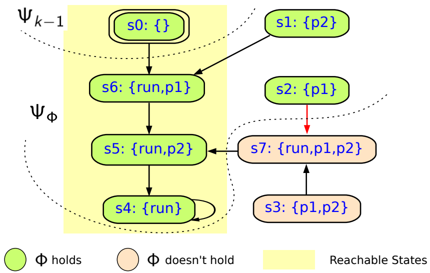
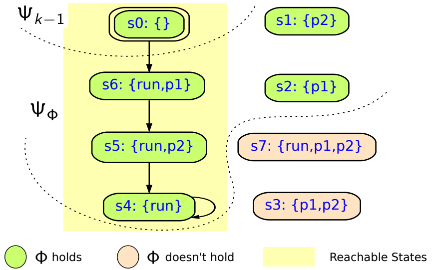
Main Idea: Modify Transition Relation to generate less CTIs

Original Transition Relation:



s_2 has successor s_7 violating Φ

Enhanced Transition Relation:



s_2 has no successor

$\Rightarrow$ remove transitions from unreachable states by **control-flow invariants**

Control-flow Invariants by static Analysis

Control-flow can never be active at both substatements of sequences and conditional statements:

```
module CfSeq(){
    p1: pause;
    p2: pause;
}
```

$$\neg(\mathrm{p1} \wedge \mathrm{p2})$$

Control-flow Invariants by static Analysis

Control-flow can never be active at both substatements of sequences and conditional statements:

```
module Ite(){
  mem bool i;
  if (i) {
    p1: pause;
  } else {
    q1: pause;
  }
}
```

$$\neg(p1 \wedge q1)$$

Control-flow Invariants by static Analysis

Control-flow can never be active at both substatements of sequences and conditional statements:

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module CfIte(){
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    p1: pause;
    p2: pause;
  } else {
    q1: pause;
    q2: pause;
  }
}
```

$$\neg(p1 \wedge p2) \wedge \neg(q1 \wedge q2) \wedge \neg((p1 \vee p2) \wedge (q1 \vee q2))$$

Control-flow Invariants by static Analysis

```
module CfIte(){
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}
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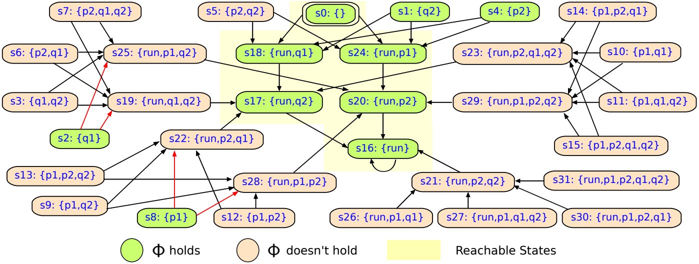

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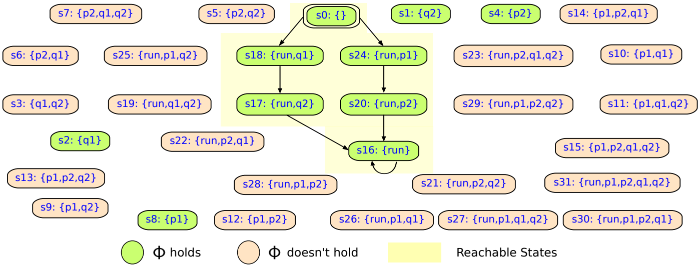


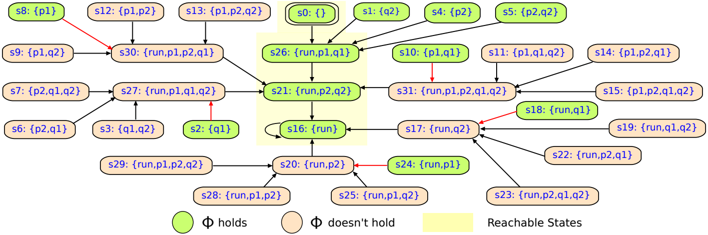
with control-flow invariant by static analysis:

$\neg(p1 \wedge p2) \wedge \neg(q1 \wedge q2) \wedge \neg((p1 \vee p2) \wedge (q1 \vee q2))$

Control-flow Invariants by symbolic Analysis

```
module CfPar(){
  {
    p1: pause;
    p2: pause;
  } ||
  {
    q1: pause;
    q2: pause;
  }
}
```

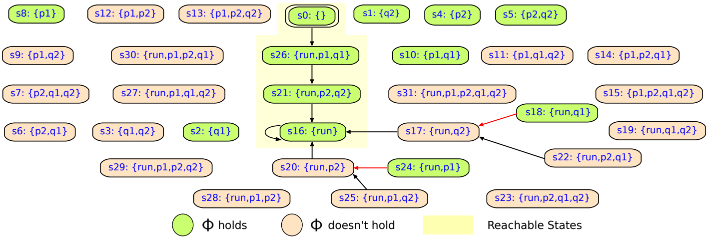
Original Transition Relation:



Control-flow Invariants by symbolic Analysis

```
module CfPar(){
  {
    p1: pause;
    p2: pause;
  } ||
  {
    q1: pause;
    q2: pause;
  }
}
```

Enhanced Transition Relation:



with control-flow invariant by static analysis:

$\neg(p1 \wedge p2) \wedge \neg(q1 \wedge q2)$

Control-flow Invariants by symbolic Analysis

Symbolic traversal of the state space of the control-flow system:

```
module CfPar(){
  {
    p1: pause;
    p2: pause;
  } ||
  {
    q1: pause;
    q2: pause;
  }
}
```

$$\neg(p1 \wedge p2) \wedge \neg(q1 \wedge q2) \wedge \neg((p1 \vee p2) \wedge (q1 \vee q2))$$

Control-flow Invariants by symbolic Analysis

```
module CfPar(){
  {
    p1: pause;
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  {
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  }
}
```
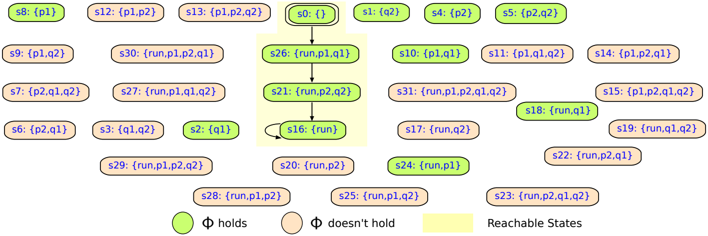

Enhanced Transition Relation:



with control-flow invariant by symbolic analysis:

$$\neg(p1 \wedge p2) \wedge \neg(q1 \wedge q2) \wedge \neg((p1 \vee p2) \wedge (q1 \vee q2))$$

Summary

Control-flow Guided PDR for Imperative Synchronous Programs

- two methods for reachable control-flow states computation
  - linear-time static analysis
  - symbolic reachability analysis

  $\Rightarrow$ different precision and runtime complexities
- enhanced transition relation makes PDR more efficient

  $\Rightarrow$ save arbitrarily many incrementation steps of PDR